Answers to Coursebook questions – Chapter 5.7

1 The flux is increasing at a constant rate so the induced emf is constant. It equals the slope, which is 2.0 V, giving the graph in the answers in the textbook.

2 The flux is not changing in the first 4 s, so the induced emf is zero. In the next 2 s the slope and hence the emf is constant at 2 V. In the last 4 s the slope is 1 V. This gives the graph in the answers in the textbook.

3 **a** In the first 4 s the emf is constant at 6 V and so the flux is increasing at a constant rate. We have a straight line graph with slope 6. In the next 2 s the emf is zero, which means that the flux is constant. Similarly, in the last 4 the emf is constant so the flux is increasing at a constant rate, i.e. the flux–time graph is a straight line with slope 12. A possible graph is shown in the answers (see page 806 in *Physics for the IB Diploma*).

b The answer is not unique because there are many straight lines with the slopes given above – we just don't know the value of the flux, just its rate of change.

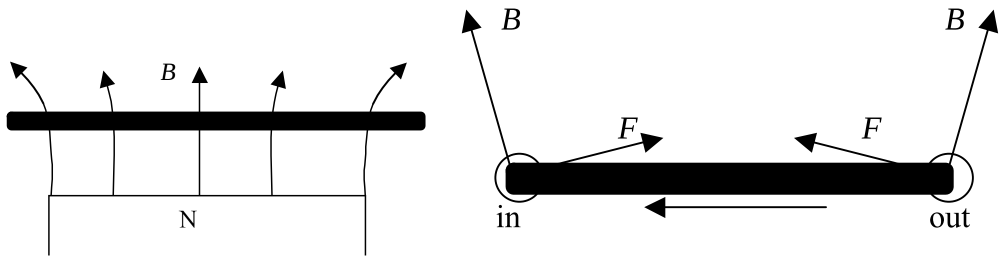
4 The magnetic field created by the outer solenoid is directed into the smaller coil. Since the current is increasing, the flux is increasing. By Lenz's law the induced current must oppose the change, i.e. it must decrease the flux. This can be done by having the induced current create a magnetic field directed out of the page. The current must then be counter-clockwise.

5 **a** Looking down from above the ring, we see that the magnetic field is directed towards us and the flux is increasing. So the induced current must produce a magnetic field directed away from us. By the right-hand rule for the magnetic field direction, the current must be clockwise. As the ring moves away from the magnet we see the magnetic field coming towards us and the flux is decreasing. So we must produce a current whose magnetic field comes towards us and so the current must be counter-clockwise. When the ring is half-way down the length of the magnet the current must be zero.

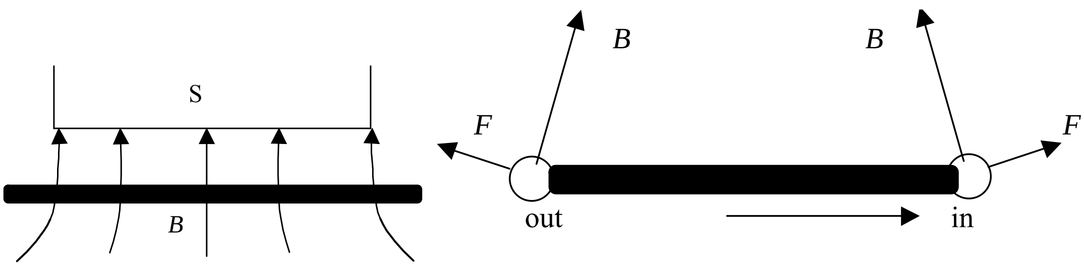
b The magnetic field we see now is directed away from us. So the induced current must create a magnetic field coming towards us and so the current is counter-clockwise. As the ring leaves, the flux is decreasing, the field is going away from us and the current must produce its own magnetic field away from us, i.e. the current is clockwise. Half-way it must be zero.

6 This is answered in **Q5**.

7 **a** The diagram shows an edge-on view of the ring as it approaches the north pole of the magnet. The forces on two diametrically opposite points on the ring are as shown. The net force of these two forces is upwards. The same is true for any other two diametrically opposite points and so the net magnetic force on the ring is vertically upwards, making the ring fall slower than in free fall.



b As the ring moves away from the south pole the diagram is:



Therefore the net force is again upwards.

8 An electron in the rod is moving downwards and since the magnetic field is into the page the magnetic force on the electron will be directed to the left. Hence the left end will be negatively charged. The electrons that have moved to the left have left the right end of the rod positively charged.

9 **a** The magnetic field at the position of the loop is coming out of the page and is increasing. Hence the flux is increasing. To oppose this increase the induced current must produce a magnetic field into the page and so the current must be counter-clockwise.

b The magnetic field at the position of the loop is coming out of the page and is decreasing. Hence the flux is decreasing. To oppose this decrease the induced current must produce a magnetic field out of the page and so the current must be clockwise.

10 The magnetic field of the large coil is $B = \mu_0 \frac{N_1 I}{L}$. The flux linkage in the smaller coil is therefore $\Phi = N_2 BA = N_2 \mu_0 \frac{N_1 I}{L} \pi r^2$. The induced emf is the rate of change of flux linkage, i.e. $\frac{\Delta\Phi}{\Delta t} = N_2 \mu_0 \frac{N_1}{L} \pi r^2 \frac{\Delta I}{\Delta t} = 200 \times 4\pi \times 10^{-7} \times \frac{10^3}{0.20} \times \pi \times 0.01^2 \times 150 = 0.059\ \text{V}$.

11 See **Q15**. It is DC.

12 The flux in the loop is increasing and so there will be an emf and a current in the loop. By Lenz's law, the magnetic field of the induced current will be directed out of the page, and hence the current will be counter-clockwise. The force on the movable rod is thus directed to the right. (Note: this could have been guessed since by moving the rod to the right we decrease the area and hence the flux. This is what must happen to oppose the change in flux, which is an increase since the field is increasing.)

13 **a** The flux is $\phi = BLx\cos\theta$.

b

$$\begin{aligned}\frac{d\phi}{dt} &= BL\frac{dx}{dt}\cos\theta \\ &= BLv\cos\theta\end{aligned}.$$

c $F = \dfrac{B^2L^2v\cos\theta}{R}$.

d $mg\sin\theta = \dfrac{B^2L^2v\cos\theta}{R} \Rightarrow v = \dfrac{mgR\tan\theta}{B^2L^2}$. $m\dfrac{\mathrm{d}v}{\mathrm{d}t} = mg\sin\theta - \dfrac{B^2L^2v\cos\theta}{R}$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g\sin\theta - \frac{B^2L^2v\cos\theta}{mR}$$

$$\frac{\mathrm{d}v}{g\sin\theta - \frac{B^2L^2v\cos\theta}{mR}} = \mathrm{d}t$$

$$\int \frac{\mathrm{d}v}{g\sin\theta - \frac{B^2L^2v\cos\theta}{mR}} = \int \mathrm{d}t$$

$$-\frac{mR}{B^2L^2\cos\theta}\ln\left(g\sin\theta - \frac{B^2L^2v\cos\theta}{mR}\right) = t + C$$

$$-\frac{mR}{B^2L^2\cos\theta}\ln\left(g\sin\theta - \frac{B^2L^2v\cos\theta}{mR}\right) = t - \frac{mR}{B^2L^2\cos\theta}\ln\left(g\sin\theta\right)$$

$$-\frac{mR}{B^2L^2\cos\theta}\ln\left(\frac{g\sin\theta - \frac{B^2L^2v\cos\theta}{mR}}{g\sin\theta}\right) = t$$

$$-\frac{mR}{B^2L^2\cos\theta}\ln\left(1 - \frac{B^2L^2v\cos\theta}{mgR\sin\theta}\right) = t$$

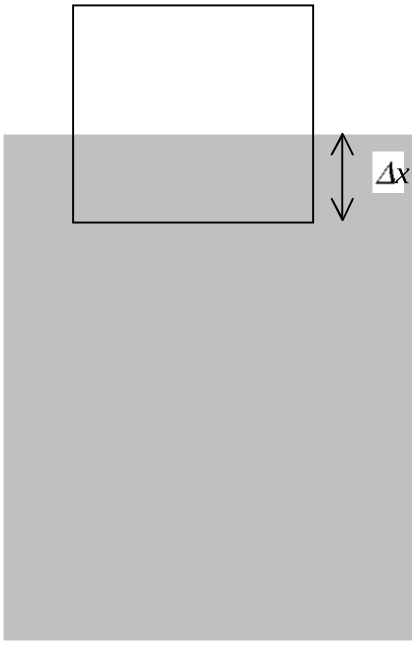
$$\ln\left(1 - \frac{B^2L^2v\cos\theta}{mgR\sin\theta}\right) = -\frac{B^2L^2\cos\theta}{mR}t$$

$$1 - \frac{B^2L^2v\cos\theta}{mgR\sin\theta} = e^{-\frac{B^2L^2\cos\theta}{mR}t}$$

$$v = \frac{mgR\tan\theta}{B^2L^2}\left(1 - \mathrm{e}^{-\frac{B^2L^2\cos\theta}{mR}t}\right)$$

e We see that as t increases the velocity tends to $\dfrac{mgR\tan\theta}{B^2L^2}$ as found before.

14 a



The flux when a length Δx of the loop has entered the region of magnetic field is $\Delta\phi = Ba\Delta x$ and so the rate of change of flux is $\frac{\Delta\phi}{\Delta t} = Ba\frac{\Delta x}{\Delta t} = Bav$. This is the induced emf and hence the current is $I = \frac{V}{R} = \frac{Bav}{R}$. The flux is increasing and the magnetic field is directed out of the page, so the induced current must produce a magnetic field directed into the page, i.e. the current must be clockwise.

b The magnetic force on the lower side of the loop is directed upwards and equals $BIa = \frac{B^2a^2v}{R}$.

Hence the net force on the loop downwards is $mg - \frac{B^2a^2v}{R}$.

Thus $ma = mg - \frac{B^2a^2v}{R} \Rightarrow a = g - \frac{B^2a^2v}{mR}$.

c We must solve $\dfrac{dv}{dt} = g - \dfrac{B^2a^2v}{mR}$, i.e.

$$\frac{\mathrm{d}v}{g - \dfrac{B^2a^2v}{mR}} = \mathrm{d}t$$

$$\int \frac{\mathrm{d}v}{g - \dfrac{B^2a^2v}{mR}} = \int \mathrm{d}t$$

$$-\frac{mR}{B^2a^2}\ln\left(g - \frac{B^2a^2v}{mR}\right) = t + C$$

When $t = 0$, $v = 0$ and so $-\dfrac{mR}{B^2a^2}\ln(g) = C$, giving

$$-\frac{mR}{B^2a^2}\ln\left(\frac{g - \dfrac{B^2a^2v}{mR}}{g}\right) = t$$

$$\ln\left(1 - \frac{B^2a^2v}{mgR}\right) = -\frac{B^2a^2}{mR}t$$

$$1 - \frac{B^2a^2v}{mgR} = \mathrm{e}^{-\frac{B^2a^2}{mR}t}$$

$$\frac{B^2a^2v}{mgR} = 1 - \mathrm{e}^{-\frac{B^2a^2}{mR}t}$$

$$v = \frac{mgR}{B^2a^2}\left(1 - \mathrm{e}^{-\frac{B^2a^2}{mR}t}\right)$$

d To find the units of $\frac{mgR}{B^2a^2}$ we need to replace the unfamiliar units of magnetic field and resistance. We know from $F = BIL$ that $\mathrm{T} = \mathrm{N\ A^{-1}\ m^{-1}}$ and from $R = \frac{V}{I}$ that $\Omega = \mathrm{V\ A^{-1}}$. Hence

$$\left[\frac{mgR}{B^2a^2}\right] = \frac{\mathrm{kg\ m\ s^{-2}\ V\ A^{-1}}}{\mathrm{N^2\ A^{-2}\ m^{-2}\ m^2}} = \frac{\mathrm{kg\ m\ s^{-2}\ V\ A}}{\mathrm{N^2}} = \frac{\mathrm{V\ A}}{\mathrm{N}} = \frac{\mathrm{W}}{\mathrm{N}} = \frac{\mathrm{N\ m\ s^{-1}}}{\mathrm{N}} = \mathrm{m\ s^{-1}}$$

and

$$\left[\frac{mR}{B^2a^2}\right] = \frac{\mathrm{kg\ V\ A^{-1}}}{\mathrm{N^2\ A^{-2}\ m^{-2}\ m^2}} = \frac{\mathrm{kg\ V\ A}}{\mathrm{N^2}} = \frac{\mathrm{kg\ W}}{\mathrm{N^2}} = \frac{\mathrm{kg\ N\ m\ s^{-1}}}{\mathrm{N^2}} = \frac{\mathrm{kg\ m\ s^{-1}}}{\mathrm{N}} = \frac{\mathrm{kg\ m\ s^{-1}}}{\mathrm{kg\ m\ s^{-2}}} = \mathrm{s}$$

Of course, having found that $\left[\frac{mgR}{B^2a^2}\right] = \mathrm{m\ s^{-1}}$ then it follows that $\left[\frac{mR}{B^2a^2}\right] = \frac{\mathrm{m\ s^{-1}}}{\mathrm{m\ s^{-2}}} = \mathrm{s}$.

15 a By considering the magnetic forces on electrons we see that electrons will tend to move towards the centre, leaving the rim positively charged and so at higher potential.

b The induced emf is $\frac{1}{2}\omega r^2 B$ and so the current is $\frac{\omega r^2 B}{2R}$.

c The electrons move towards the centre and away from the rim, so the current is clockwise.

d The power dissipated is $P = \frac{V^2}{R} = \frac{(\omega r^2 B)^2}{4R}$ as required.